

Boundary problems for the one-dimensional kinetic equation with frequency of collisions, affine depending on the module velocity

A. L. Bugrimov¹, A. V. Latyshev² and A. A. Yushkanov³

*Faculty of Physics and Mathematics,
Moscow State Regional University, 105005,
Moscow, Radio str., 10A*

Abstract

For the one-dimensional linear kinetic equation analytical solutions of problems about temperature jump and weak evaporation (condensation) over flat surface are received. The equation has integral of collisions BGK (Bhatnagar, Gross and Krook) and frequency of collisions of molecules, affine depending on the module molecular velocity.

Key words: kinetic equation, frequency of collisions, preservation laws, separation of variables, characteristic equation, dispersion equation, eigenfunctions, analytical solution.

PACS numbers: 05.60.-k Transport processes, 51.10.+y Kinetic and transport theory of gases,

Introduction

In work [1] the linear one-dimensional kinetic equation with integral of collisions BGK (Bhatnagar, Gross and Krook) and frequency of collisions, affine depending on the module velocity of molecules has been entered.

In [1] the theorem about structure of general solution of the entered equation has been proved.

¹*fakul – fm@mgou.ru*

²*avlatyshev@mail.ru*

³*yushkanov@inbox.ru*

In the present work which is continuation [1], exact solutions of the problem about temperature jump and weak evaporation (condensation) in the rarefied gas are received. These two problems following [2] we will name the generalized Smoluchowsky' problem , or simply the Smoluchowsky problem.

Let us stop on history of exclusively analytical solutions of the generalized Smoluchowsky' problem.

For simple (one-nuclear) rarefied gas with a constant frequency of collisions of molecules the analytical solution of the generalized of Smoluchowsky' problems it is received in [3].

In [4] the generalized of Smoluchowsky' problem was analytically solved for simple rarefied gas with frequency of collisions the molecules, linearly depending on the module of molecular velocity. In [5] the problem about strong evaporation (condensation) with constant frequency of collisions has been analytically solved.

Let us notice, that for the first time the problem about temperature jump with frequency of collisions of molecules, linearly depending on the module molecular velocity, was analytically solved by Cassel and Williams in work [6] in 1972.

Then in works [7, 8, 9] the generalized Smoluchowsky' problem also analytical solution for case of multinuclear (molecular) gases has been received.

In works [10, 11, 12] the problem about behaviour of the quantum Boze-gas at low temperatures (similar to the temperature jump problem for electrons in metal) is considered. We used the kinetic equation with excitation fonons agrees to N.N. Bogolyubov.

In works [13, 14] the problem about temperature jump for electrons of degenerate plasmas in metal has been solved.

In work [15] the analytical solution of the Smoluchowsky' problem for quantum gases it has been received.

In work of Cercignani and Frezzotti [16] the Smoluchowsky' problem

it was considered with use of the one-dimensional kinetic equations. The full analytical solution of Smoluchowsky' problem with use of Cercignani—Frezotti equation it has been received in work [17].

At the same time there is an unresolved problem about temperature jump and concentration with use of the BGK-equation with arbitrary dependence of frequency on velocity, in spite of on obvious importance of the decision of a problem in similar statement.

In the present work attempt to promote in this direction is made. Here the case of the affine dependence of collision frequency on molecular velocity in models of one-dimensional gas is considered. Model of one-dimensional gas gave the good consent with the results devoted to the three-dimensional gas [17].

Let us start with statement problem. Then we will give the solution of the Smoluchowsky' problem for the one-dimensional kinetic equation with frequency of collisions, affine depending on the module of molecular velocity.

1. Statement of the problem and the basic equations

Let us begin with the general statement. Let gas occupies half-space $x > 0$. The surface temperature T_s and concentration of sated steam of a surface n_0 are set. Far from a surface gas moves with some velocity u , being velocity of evaporation (or condensation), also has the temperature gradient

$$g_T = \left(\frac{d \ln T}{dx} \right)_{x=+\infty}.$$

It is necessary to define jumps of temperature and concentration depending on velocity and temperature gradient.

In a problem about weak evaporation it is required to define temperature and concentration jumps depending on velocity, including a temperature gradient equal to zero, and velocity of evaporation (condensation) is enough small. The last means, that

$$u \ll v_T.$$

Here v_T is the heat velocity of molecules, having order of sound velocity order,

$$v_T = \frac{1}{\sqrt{\beta_s}}, \quad \beta_s = \frac{m}{2k_B T_s},$$

m is the mass of molecule, k_B is the Boltzmann constant.

In the problem about temperature jump it is required to define temperature and concentration jumps depending on a temperature gradient, thus evaporation (condensation) velocity it is considered equal to zero, and the temperature gradient is considered as small. It means, that

$$lg_T \ll 1, \quad l = \tau v_T, \quad \tau = \frac{1}{\nu_0},$$

where l is the mean free path of gas molecules, τ is the mean relaxation time, i.e. time between two consecutive collisions of molecules.

Let us unite both problems (about weak evaporation (condensation) and temperature jump) in one. We will assume that the gradient of temperature is small (i.e. relative difference of temperature on length of mean free path is small) and the velocity of gas in comparison with sound velocity is small. In this case the problem supposes linearization and distribution function it is possible to search in the form

$$f(x, v) = f_0(v)(1 + h(x, v)),$$

where

$$f_0(v) = n_s \left(\frac{m}{2\pi k_B T_s} \right)^{1/2} \exp \left[- \frac{mv^2}{2k_B T_s} \right]$$

is the absolute Maxwellian.

We take the linear kinetic equation which has been written down rather functions $h(x, v)$, with integral of collisions of relaxation type, in [1] integral of collisions BGK named also (Bhatnagar, Gross and Krook), and having the following form

$$v \frac{\partial h}{\partial x} = \nu(v) \left[A_0 + A_1 \frac{v}{v_T} + A_2 \left(\frac{v^2}{v_T^2} - \beta \right) - h(x, v) \right]. \quad (1.1)$$

Here A_α ($\alpha = 0, 1, 2$) is the any constants, subject to definition from laws of preservation of number of particles (numerical density), an momentum and energy, $\nu(v)$ is the collision frequency affine depending on module molecular velocity,

$$\nu(v) = \nu_0 \left(1 + \sqrt{\pi} a \sqrt{\frac{m}{2k_B T_s}} |v| \right),$$

a is the arbitrary positive paramater, $0 \leq a < +\infty$.

Strictly speaking, in such model is necessary to take frequency of collisions in the form

$$\nu(v) = \nu_0 \left(1 + \sqrt{\pi} a \sqrt{\frac{m}{2k_B T_s}} |v - u_0(x)| \right),$$

where $v - u_0(x)$ is the velocity of a molecule in system of coordinates, concerning which gas is rested in the given point x , v is the velocity of a molecule in laboratory system of coordinates, $u_0(x)$ is the mass velocity of gas in a point x in laboratory system coordinates. We will consider further, that in linear statement $|u_0(x)| \ll v_T$.

The right part of the equation (1.1) is the linear integral of collisions, expanded on collision invariants

$$\begin{aligned} \psi_0(v) &= 1, \\ \psi_1(v) &= \sqrt{\frac{m}{2k_B T_s}} v, \\ \psi_2(v) &= \frac{mv^2}{2k_B T_s} - \beta. \end{aligned}$$

Let us pass in the equation (1.1) to dimensionless velocity

$$C = \sqrt{\beta} v = \frac{v}{v_T}$$

and dimensionless coordinate

$$x' = \nu_0 \sqrt{\frac{m}{2k_B T_s}} x = \frac{x}{l}$$

The variable x' let us designate again through x .

In the dimensionless variables we will rewrite the equation (1.1) in the form

$$C \frac{\partial h}{\partial x} = (1 + \sqrt{\pi}a|C|) \left[l_0[h] + 2Cl_1[h] + (C^2 - \beta)l_2[h] - h(x, C) \right]. \quad (1.2)$$

The constant β is finding from an orthogonality condition of invariants $\psi_0(v)$ and $\psi_2(v)$. Orthogonality here is understood as equality to zero of scalar product with weight $\rho(C) = (1 + \sqrt{\pi}a|C|) \exp(-C^2)$

$$(f, g) = \int_{-\infty}^{\infty} \rho(C) f(C) g(C) dC.$$

From here we receive that

$$\beta = \beta(a) = \frac{2a + 1}{2(a + 1)}.$$

2. Laws of preservation and transformation of the kinetic equation

The modelling integral of collisions should satisfy to laws preservations of number of particles (numerical density), momentum and energy

$$(\psi_\alpha, M[h]) \equiv \nu_0 \int_{-\infty}^{\infty} e^{-C^2} (1 + \sqrt{\pi}a|C|) M[h] \psi_\alpha(C) dC = 0, \quad (2.1)$$

where

$$M[h] = A_0 + A_1 + (C^2 - \beta)A_2 - h(x, C).$$

From the first equation from (2.1), i.e. preservation law of number of particles $(\psi_0, M[h]) = 0$ we receive that

$$A_0 = \frac{(1, h)}{(1, 1)} = \frac{1}{\sqrt{\pi}(a + 1)} \int_{-\infty}^{\infty} e^{-C^2} (1 + \sqrt{\pi}a|C|) h(x, C) dC..$$

From second equation from (2.1), i.e. preservation law of momentum $(\psi_1, M[h]) = 0$ we receive that

$$A_1 = \frac{(C, h)}{(C, C)} = \frac{2}{\sqrt{\pi}(2a + 1)} \int_{-\infty}^{\infty} e^{-C^2} (1 + \sqrt{\pi}a|C|) C h(x, C) dC.$$

From third equation from (2.1), i.e. preservation law of energy $(\psi_2, M[h]) = 0$ we receive that

$$\begin{aligned} A_2 &= \frac{(C^2 - \beta, h)}{(C^2 - \beta, C^2 - \beta)} = \\ &= \frac{4(a+1)}{\sqrt{\pi}(4a^2 + 7a + 2)} \int_{-\infty}^{\infty} e^{-C^2} (1 + \sqrt{\pi}a|C|)(C^2 - \beta)h(x, C)dC. \end{aligned}$$

Let us return to the equation (1.2) and by means of received above equalities let us transform this equation to the form

$$\begin{aligned} &C \frac{\partial h}{\partial x} + (1 + \sqrt{\pi}a|C|)h(x, C) = \\ &= (1 + \sqrt{\pi}a|C|) \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} e^{-C'^2} (1 + \sqrt{\pi}a|C'|)q(C, C', a)h(x, C')dC'. \quad (2.2) \end{aligned}$$

Here $q(C, C', a)$ is the kernel of equation,

$$q(C, C', a) = r_0(a) + r_1(a)CC' + r_2(a)(C^2 - \beta(a))(C'^2 - \beta(a)),$$

$$r_0(a) = \frac{1}{a+1}, \quad r_1(a) = \frac{2}{2a+1}, \quad r_2(a) = \frac{4(a+1)}{4a^2 + 7a + 2}.$$

3. Derivation of boundary conditions and the formulation of boundary problem

Rectilinear substitution it is possible to check up, that the kinetic equation (2.2) has following four private solutions

$$h_0(x, C) = 1,$$

$$h_1(x, C) = C,$$

$$h_2(x, C) = C^2,$$

$$h_3(x, C) = \left(C^2 - \frac{3}{2}\right) \left(x - \frac{C}{1 + \sqrt{\pi}a|C|}\right).$$

Let us consider, that molecules are reflected from a wall purely diffusively, i.e. they are reflected from a wall with Maxwell distribution by velocities, i.e.

$$f(x, v) = f_0(v), \quad v_x > 0.$$

From here we receive for function $h(x, C)$ condition

$$h(0, C) = 0, \quad C > 0. \quad (3.1)$$

Condition (3.1) is the first boundary condition to the equation (2.2).

For asymptotic distribution of Chepman—Enskog we will search in the form of a linear combination of its partial solutions with unknown coefficients

$$\begin{aligned} h_{as}(x, C) = & A_0 + A_1 C + A_2 \left(C^2 - \frac{1}{2}\right) + \\ & + A_3 \left(C^2 - \frac{3}{2}\right) \left(x - \frac{C}{1 + \sqrt{\pi}a|C|}\right). \end{aligned} \quad (3.2)$$

We consider the distribution of number density

$$n(x) = \int_{-\infty}^{\infty} f(x, v) dv = \int_{-\infty}^{\infty} f_0(v)(1 + h(x, v)) dv = n_0 + \delta n(x).$$

Here

$$n_0 = \int_{-\infty}^{\infty} f_0(v) dv, \quad \delta n(x) = \int_{-\infty}^{\infty} f_0(v) h(x, v) dv.$$

From here we receive that

$$\frac{\delta n(x)}{n_0} = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} e^{-C^2} h(x, C) dC.$$

We denote

$$n_e = n_0 \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} e^{-C^2} (1 + h_{as}(x = 0, C)) dC.$$

From here we receive that

$$\varepsilon_n \equiv \frac{n_e - n_0}{n_0} = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} e^{-C^2} h_{as}(x=0, C) dC. \quad (3.3)$$

The quantity ε_n is the unknown jump of concentration.

Substituting (3.2) in (3.3), we find, that

$$\varepsilon_n = A_0. \quad (3.4)$$

From definition of dimensional velocity of gas

$$u(x) = \frac{1}{n(x)} \int_{-\infty}^{\infty} f(x, v) v dv$$

we receive, that in linear approximation dimensional mass velocity is equal

$$U(x) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} e^{-C^2} h(x, C) C dC.$$

Setting "far from a wall" velocity of evaporation (condensation), let us write

$$U = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} e^{-C^2} h_{as}(x, C) C dC. \quad (3.5)$$

Substituting in (3.5) distribution (3.2), we receive, that

$$A_1 = 2U + A_3 \omega(a), \quad (3.6)$$

where $\omega(a)$ is the number parameter,

$$\omega(a) = \frac{2}{\sqrt{\pi}} \int_{-\infty}^{\infty} e^{-C^2} \frac{C^2(C^2 - 3/2) dC}{1 + \sqrt{\pi} a |C|}.$$

Let us consider temperature distribution

$$T(x) = \frac{2}{kn(x)} \int_{-\infty}^{\infty} \frac{m}{2} (v - u_0(x))^2 f(x, v) dv.$$

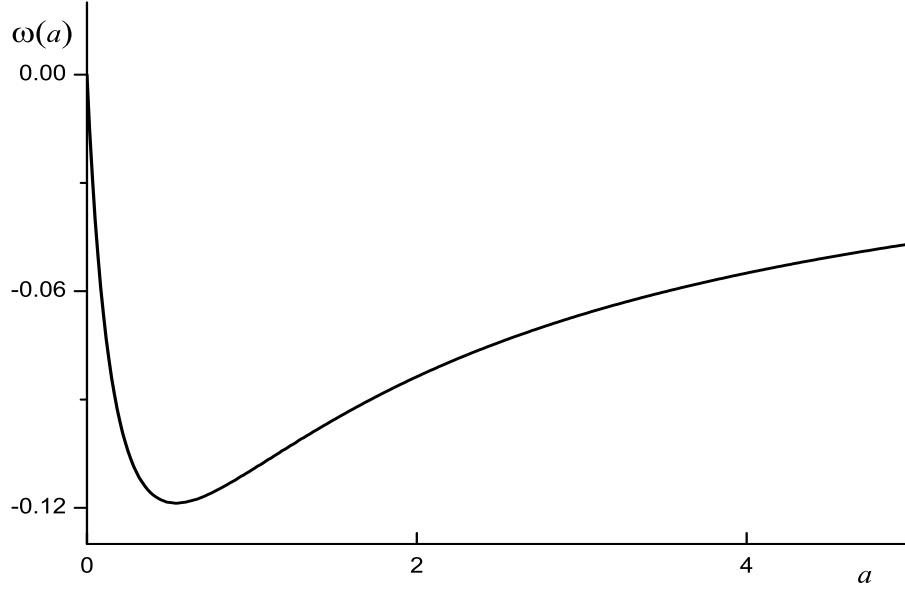


Fig. 1. Dependence of quantity $\omega = \omega(a)$ on parameter of problem a .

From here we find, that

$$\begin{aligned} \frac{\delta T(x)}{T_0} &= -\frac{\delta n}{n_0} + \frac{2}{\sqrt{\pi}} \int_0^\infty e^{-C^2} h(x, C) C^2 dC = \\ &= \frac{2}{\sqrt{\pi}} \int_0^\infty e^{-C^2} h(x, C) (C^2 - \frac{1}{2}) dC. \end{aligned}$$

From here follows, that at $x \rightarrow +\infty$ asymptotic distribution is equal

$$\frac{\delta T_{as}(x)}{T_0} = \frac{2}{\sqrt{\pi}} \int_0^\infty e^{-C^2} h_{as}(x, C) (C^2 - \frac{1}{2}) dC. \quad (3.7)$$

Setting of the gradient of temperature far from a wall means, that distribution of temperature looks like

$$T(x) = T_e + \left(\frac{dT}{dx} \right)_{x=+\infty} \cdot x = T_e + G_T x,$$

where

$$G_T = \left(\frac{dT}{dx} \right)_{+\infty}.$$

This distribution we will present in the form

$$T(x) = T_s \left(\frac{T_e}{T_s} + g_T x \right) = T_s \left(1 + \frac{T_e - T_s}{T_s} + g_T x \right), \quad x \rightarrow +\infty,$$

where

$$g_T = \left(\frac{d \ln T}{dx} \right)_{x=+\infty},$$

or

$$T(x) = T_s(1 + \varepsilon_T + g_T x), \quad x \rightarrow +\infty,$$

where

$$\varepsilon_T = \frac{T_e - T_s}{T_s}$$

is the unknown temperature jump.

From expression (3.7) is visible, that relative change of temperature far from a wall is described by linear function

$$\frac{\delta T_{as}(x)}{T_s} = \frac{T(x) - T_s}{T_s} = \varepsilon_T + g_T x, \quad x \rightarrow +\infty \quad (3.8)$$

Substituting (3.2) in (3.7), we receive, that

$$\frac{\delta T_{as}(x)}{T_s} = A_2 + A_3 x. \quad (3.10)$$

Comparing (3.7) and (3.10), we find

$$A_2 = \varepsilon_T, \quad A_3 = g_T.$$

So, asymptotic function of Chepmen—Enskog' distribution is constructed

$$h_{as}(x, C) = \varepsilon_n + \varepsilon_T + (2U + \omega(a)g_T)C + \left(C^2 - \frac{3}{2} \right) \left[\varepsilon_T + g_T \left(x - \frac{C}{1 + \sqrt{\pi}a|C|} \right) \right].$$

Now we will formulate the second boundary condition to the equation (2.2)

$$h(x, C) = h_{as}(x, C) + o(1), \quad x \rightarrow +\infty. \quad (3.11)$$

Now we will formulate the basic boundary problem, which is generalized Smoluchowsky' problem. This problem consists in finding of such solution of the kinetic equation (2.2) which satisfies to boundary conditions (3.1) and (3.11).

4. Transformation of boundary problem

In the equation (2.2) we will carry out variable replacement $\sqrt{\pi}a \rightarrow a$ also we will write down the received equation in the form

$$\begin{aligned} & \frac{C}{1+a|C|} \frac{\partial h}{\partial x} + h(x, C) = \\ & = \int_{-\infty}^{\infty} e^{-C'^2} (1+a|C'|) q(C, C', a) h(x, C') dC'. \end{aligned} \quad (4.1)$$

In this equation $q(C, C', a)$ is the kernel of equation,

$$q(C, C', a) = r_0(a) + r_1(a)CC' + r_2(a)(C^2 - \beta(a))(C'^2 - \beta(a)),$$

where

$$r_0(a) = \frac{1}{a + \sqrt{\pi}}, \quad r_1(a) = \frac{2}{2a + \sqrt{\pi}}, \quad r_2(a) = \frac{4(a + \sqrt{\pi})}{4a^2 + 7\sqrt{\pi}a + 2\pi},$$

$$\beta(a) = \frac{1}{2} \frac{2a + \sqrt{\pi}}{a + \sqrt{\pi}}.$$

Let us make in the equation (4.1) replacement of a variable $C = C(\mu)$, $C' = C(\mu')$, where

$$C(\mu) = \frac{\mu}{1 - a|\mu|}, \quad |\mu| < \alpha, \quad \alpha = \frac{1}{a}.$$

Let us designate function $h(x, C(\mu))$ again through $h(x, \mu)$. The equation (4.1) passes in the following equation, standard for the transport theory

$$\mu \frac{\partial h}{\partial x} + h(x, \mu) = \int_{-\alpha}^{\alpha} \rho(\mu') q(\mu, \mu') h(x, \mu') d\mu', \quad (4.2)$$

where

$$\begin{aligned}\rho(\mu') &= \exp \left[- \left(\frac{\mu'}{1 - a|\mu'|} \right)^2 \right] \frac{1}{(1 - a|\mu'|)^3}, \\ q(\mu, \mu') &= r_0(a) + r_1(a) \frac{\mu}{1 - a|\mu|} \frac{\mu'}{1 - a|\mu'|} + \\ &+ r_2(a) \left[\left(\frac{\mu}{1 - a|\mu|} \right)^2 - \beta(a) \right] \left[\left(\frac{\mu'}{1 - a|\mu'|} \right)^2 - \beta(a) \right].\end{aligned}$$

Let us notice, that on the ends of the intervals of integration we have

$$\rho(\pm\alpha) = 0,$$

and, besides,

$$\lim_{\mu \rightarrow \pm\alpha} \rho(\mu) C^n(\mu) = 0$$

for any natural n .

Boundary conditions (3.1) and (3.11) after variable replacement $C = C(\mu)$ pass in the following

$$h(0, \mu) = 0, \quad 0 < \mu < \alpha, \quad (4.3)$$

and

$$h(x, \mu) = h_{as}(x, \mu) + o(1), \quad x \rightarrow +\infty, \quad (4.4)$$

where

$$\begin{aligned}h_{as}(x, \mu) &= \varepsilon_n + \varepsilon_T + (2U + g_T \omega(a)) C(\mu) + \\ &+ \left(C^2(\mu) - \frac{3}{2} \right) \left[\varepsilon_T + g_T(x - \mu) \right].\end{aligned} \quad (4.5)$$

In (4.5) the designation is entered

$$\omega(a) = \frac{2}{\sqrt{\pi}} \int_{-\alpha}^{\alpha} e^{-C^2(\mu)} C^2(\mu) \left[C^2(\mu) - \frac{3}{2} \right] \frac{d\mu}{1 - a|\mu|}.$$

Let us solve further a boundary problem (4.2) – (4.4).

5. Eigenfunction and eigenvalues

Seperation of variables in the equation (4.2), taken in the form

$$h_\eta(x, \mu) = \exp\left(-\frac{x}{\eta}\right)\Phi(\eta, \mu), \quad \eta \in \mathbb{C}, \quad (5.1)$$

transforms equation (5.1) to characteristic equation

$$(\eta - \mu)\Phi(\eta, \mu) = \eta Q(\eta, \mu), \quad \eta, \mu \in (-\alpha, +\alpha), \quad (5.2)$$

where

$$Q(\eta, \mu) = r_0(a)n_0(\eta) + r_1(a)C(\mu)n_1(\eta) + \\ + r_2(a)\left(C^2(\eta) - \beta(a)\right)\left(C^2(\mu) - \beta(a)\right).$$

Here

$$n_\alpha(\eta) = \int_{-\alpha}^{\alpha} \Phi(\eta, \mu) C^\alpha(\mu) \rho(\mu) d\mu, \quad \alpha = 0, 1, 2, \quad (5.3)$$

is the zero, first and second moments of eigen function with weight $\rho(\mu)$.

Eigen functions of the continuous spectrum filling by the continuous fashion an interval $(-\alpha, \alpha)$, we find [18] in space of the generalized functions

$$\Phi(\eta, \mu) = \eta Q(\eta, \mu) P \frac{1}{\eta - \mu} + g(\eta) \delta(\eta - \mu), \quad \eta \in (-\alpha, \alpha). \quad (5.4)$$

Here $g(\eta)$ is the unknown function, defined from equations (5.3), Px^{-1} is the distribution, meaning principal value of integral by integration x^{-1} , $\delta(x)$ is the Dirac delta-function.

Let us substitute eigen functions (5.4) in normalization equalities (5.3). We will receive the following system of the dispersion equations

$$n_\alpha(\eta) + \eta \int_{-\alpha}^{\alpha} Q(\eta, \mu) C^\alpha(\mu) \rho(\mu) \frac{d\mu}{\mu - \eta} = g(\eta) \rho(\eta) C^\alpha(\eta), \quad (5.5)$$

$\alpha = 0, 1, 2$.

We denote

$$t_n(\eta) = \eta \int_{-\alpha}^{\alpha} C^n(\mu) \frac{\rho(\mu) d\mu}{\mu - \eta}, \quad n = 0, 1, 2, 3, 4.$$

Now system of the dispersion equations (5.5) it is possible transform to the form

$$\begin{aligned} & n_\alpha(\eta) + r_0(a)n_0(\eta)t_\alpha(a) + r_1(a)n_1(\eta)t_1(\eta) + \\ & + (n_2(\eta) - \beta(a)n_0(\eta))(t_{\alpha+2}(\eta) - \beta(a)t_\alpha(\eta)) = g(\eta)\rho(\eta)C^\alpha(\eta), \end{aligned} \quad (5.6)$$

where $\alpha = 0, 1, 2$.

Let us write down the equations (5.6) in the vector form

$$\Lambda(\eta)n(\eta) = g(\eta)\rho(\eta) \begin{bmatrix} 1 \\ C(\eta) \\ C^2(\eta) \end{bmatrix}. \quad (5.7)$$

Here $\Lambda(\eta)$ is the dispersion matrix-function with elements

$$\lambda_{ij}(\eta) \quad (i, j = 1, 2, 3),$$

$n(\eta)$ is the normalization vector with elements $n_\alpha(\eta)$ ($\alpha = 0, 1, 2$).

Elements of the dispersion matrix in the explicit form will more low be necessary

$$\lambda_{11}(z) = 1 + \left[r_2(a) + \beta^2(a)r_2(a) \right] t_0(z) - \beta(a)r_2(a)t_2(z),$$

$$\lambda_{12}(z) = r_1(a)t_1(z),$$

$$\lambda_{13}(z) = r_2(a) \left[-\beta(a)t_0(z) + t_2(z) \right],$$

$$\lambda_{21}(z) = \left[r_0(a) + \beta^2(a)r_2(a) \right] t_1(z) - \beta(a)r_2(a)t_3(z),$$

$$\lambda_{22}(z) = 1 + r_1(a)t_3(z),$$

$$\lambda_{23}(z) = r_2(a) \left[-\beta(a)t_1(z) + t_3(z) \right],$$

$$\begin{aligned}
\lambda_{31}(z) &= \left[r_0(a) + \beta^2(a)r_2(a) \right] t_2(z) - \beta(a)r_2(a)t_4(z), \\
\lambda_{32}(z) &= r_1(a)t_3(z), \\
\lambda_{33}(z) &= 1 + r_2(a) \left[-\beta(a)t_2(z) + t_4(z) \right].
\end{aligned}$$

We introduce the dispersion function $\lambda(z)$, $\lambda(z) = \det \Lambda(z)$. In the explicit form we have

$$\begin{aligned}
\lambda(z) &= \lambda_{11}(z)\lambda_{22}(z)\lambda_{33}(z) + r_1(a)t_3(z)\lambda_{13}(z)\lambda_{21}(z) + \\
&+ r_1(a)t_1(z)\lambda_{31}(z)\lambda_{23}(z) - \lambda_{13}(z)\lambda_{22}(z)\lambda_{31}(z) - \\
&- r_1(a)t_3(z)\lambda_{11}(z)\lambda_{23}(z) - r_1(a)t_1(z)\lambda_{21}(z)\lambda_{33}(z).
\end{aligned}$$

From vector equation (5.7) we find

$$n_\alpha(\eta) = g(\eta)\rho(\eta)\frac{\Lambda_\alpha(\eta)}{\lambda(\eta)}, \quad \alpha = 0, 1, 2, \quad (5.8)$$

where $\Lambda_\alpha(\eta)$ is the determinant received from determinant of system (5.6) by replacement in it α -th column by the column from free members of this system. We will write out these determinants in the explicit form

$$\begin{aligned}
\Lambda_0(z) &= \Lambda_{11}(z) - C(z)\Lambda_{21}(z) + C^2(z)\Lambda_{31}(z) = \lambda_{22}(z)\lambda_{33}(z) - \\
&- r_1(a)t_3(z)\lambda_{23}(z) - C(z)r_1(a) \left[t_1(z)\lambda_{33}(z) - t_2(z)\lambda_{13}(z) \right] + \\
&+ C^2(z) \left[r_1(a)t_1(z)\lambda_{23}(z) - \lambda_{22}(z)\lambda_{13}(z) \right],
\end{aligned}$$

$$\begin{aligned}
\Lambda_1(z) &= \Lambda_{12}(z) + C(z)\Lambda_{22}(z) - C^2(z)\Lambda_{32}(z) = -\lambda_{21}(z)\lambda_{33}(z) + \\
&+ \lambda_{31}(z)\lambda_{33}(z) + C(z) \left[\lambda_{11}(z)\lambda_{33}(z) - \lambda_{31}(z)\lambda_{13}(z) \right] - \\
&- C^2(z) \left[\lambda_{11}(z)\lambda_{23}(z) - \lambda_{21}(z)\lambda_{13}(z) \right],
\end{aligned}$$

$$\begin{aligned}
\Lambda_2(z) &= \Lambda_{31}(z) - C(z)\Lambda_{32}(z) + C^2(z)\Lambda_{33}(z) = \\
&= r_1(a)t_3(z)\lambda_{21}(z) - \lambda_{31}(z)\lambda_{22}(z) - C(z)r_1(a)\left[t_3(z)\lambda_{11}(z) - t_1(z)\lambda_{33}(z)\right] + \\
&\quad + C^2(z)\left[\lambda_{11}(z)\lambda_{22}(z) - r_1(a)t_1(z)\lambda_{21}(z)\right].
\end{aligned}$$

Here $\Lambda_{ij}(z)$ is the minor of element $\lambda_{ij}(z)$.

By means of equalities (5.8) we will transform equality for $Q(\eta, \mu)$ to the form

$$Q(\eta, \mu) = \tilde{Q}(\eta, \mu) \frac{g(\eta)}{\lambda(\eta)} \rho(\eta), \quad (5.9)$$

where

$$\begin{aligned}
\tilde{Q}(\eta, \mu) &= r_0(a)\Lambda_0(\eta) + r_1(a)C(\mu)\Lambda_1(\eta) + \\
&\quad + r_2(a)\left[C^2(\mu) - \beta(a)\right]\left[\Lambda_2(\eta) - \beta(a)\Lambda_0(\eta)\right].
\end{aligned}$$

By means of equality (5.9) we will transform expression (5.4) for eigen functions

$$\Phi(\eta, \mu) = \tilde{\Phi}(\eta, \mu)g(\eta), \quad (5.10)$$

where

$$\tilde{\Phi}(\eta, \mu) = \eta \frac{\tilde{Q}(\eta, \mu)}{\lambda(\eta)} \rho(\eta) P \frac{1}{\eta - \mu} + \delta(\eta - \mu). \quad (5.11)$$

From equality (5.10) it is visible, that eigen functions are defined accurate within to coefficient – any function $g(\eta)$, identically not equal to zero. Owing to uniformity of the initial kinetic equation it is possible to consider this function identically equal to unit ($g(\eta) \equiv 1$) and further in quality eigen function corresponding to continuous spectrum, it is possible to consider the functions defined by equality (5.11). Apparently from the solution of the characteristic equation, continuous spectrum of the characteristic equation is the set

$$\sigma_c = \{\eta : -\alpha < \eta < +\alpha\}.$$

By definition the discrete spectrum of the characteristic equation consists of set of zero of dispersion function.

Expanding dispersion function in Laurent series in a vicinity infinitely remote point, we are convinced, that it in this point has zero of the fourth order. Applying an argument principle [19] from the theory of functions complex variable, it is possible to show, that other zero, except $z_i = \infty$, dispersion function not has. Thus, the discrete spectrum of the characteristic equations consists of one point $z_i = \infty$, multiplication factor which it is equal four,

$$\sigma_d = \{z_i = \infty\}.$$

To point $z_i = \infty$, as to the 4-fold point of discrete spectrum, corresponds following four discrete (partial) solutions of the kinetic decision (4.2)

$$h_0(x, \mu) = 1,$$

$$h_1(x, \mu) = C(\mu),$$

$$h_2(x, \mu) = C^2(\mu) - \frac{1}{2},$$

$$h_3(x, \mu) = (x - \mu) \left(C^2(\mu) - \frac{3}{2} \right).$$

Let us result formulas Sokhotsky for the difference and the sum of the boundary values of dispersion function from above and from below on the $(-\alpha, +\alpha)$

$$\lambda^+(\mu) - \lambda^-(\mu) = 2\pi i \rho(\mu) \tilde{Q}(\mu, \mu), \quad \mu \in (-\alpha, +\alpha),$$

and

$$\frac{\lambda^+(\mu) + \lambda^-(\mu)}{2} = \lambda(\mu), \quad \mu \in (-\alpha, +\alpha).$$

6. Analytical solution of boundary value problem

Here we will prove the theorem about analytical solution of the basic boundary problem (4.2) - (4.4).

THEOREM. *The boundary problem (4.2)–(4.4) has the unique solution, representable in the form of the sum linear combinations of discrete*

(partial) solutions of this equation and integral on the continuous spectrum on eigenfunctions correspond to the continuous spectrum

$$h(x, \mu) = h_{as}(x, \mu) + \int_0^\alpha \exp\left(-\frac{x}{\eta}\right) F(\eta, \mu) A(\eta) d\eta. \quad (6.1)$$

In equality (6.1) ε_n and ε_T are unknown coefficients (of discrete spectrum), U and g_T are the given values, $A(\eta)$ is the unknown function (coefficient of the continuous spectrum).

Coefficients of discrete and continuous spectra are subject to finding from boundary conditions.

Expansion (6.1) it is possible to present in classical sense

$$h(x, \mu) = \varepsilon_n + \varepsilon_T + (2U + g_T \omega) C(\mu) + \left(C^2(\mu) - \frac{3}{2}\right) \left[\varepsilon_T + \right. \\ \left. + g_T(x - \mu)\right] + e^{-x/\mu} A(\mu) + \int_0^\alpha e^{-x/\eta} \frac{\eta R(\eta, \mu) \rho(\eta) A(\eta)}{\lambda(\eta)(\eta - \mu)} d\eta. \quad (6.1')$$

PROOF. Let us substitute decomposition (6.1) in a boundary condition (4.2). We receive the integral equation

$$h_{as}(0, \mu) + \int_0^\alpha F(\eta, \mu) A(\eta) d\eta = 0, \quad 0 < \mu < \alpha.$$

In an explicit form this equation looks like

$$h_{as}(0, \mu) + \int_0^\alpha \frac{\eta R(\eta, \mu) \rho(\eta)}{\lambda(\eta)(\eta - \mu)} A(\eta) d\eta + A(\mu) = 0, \quad 0 < \mu < \alpha. \quad (6.2)$$

Let us enter auxiliary function

$$N(z) = \int_0^\alpha \frac{\eta R(\eta, z) \rho(\eta)}{\lambda(\eta)(\eta - z)} A(\eta) d\eta, \quad (6.3)$$

for which according to formulas Sohotsky we have

$$N^+(\mu) - N^-(\mu) = 2\pi i \mu \frac{R(\mu, \mu)}{\lambda(\mu)} \rho(\mu) A(\mu), \quad 0 < \mu < \alpha, \quad (6.4)$$

$$\frac{N^+(\mu) + N^-(\mu)}{2} = \int_0^\alpha \frac{\eta R(\eta, \mu) \rho(\eta)}{\lambda(\eta)(\eta - \mu)} A(\eta) d\eta, \quad 0 < \mu < \alpha. \quad (6.5)$$

Let us transform the equation (6.2) according to equalities (6.4) and (6.5)

$$h_{as}(0, \mu) + \frac{N^+(\mu) + N^-(\mu)}{2} + \lambda(\mu) \frac{N^+(\mu) - N^-(\mu)}{2\pi i \rho(\mu) R(\mu, \mu)} = 0,$$

whence

$$\begin{aligned} 2\pi i \mu \rho(\mu) R(\mu, \mu) h_{as}(0, \mu) + \pi i \mu \rho(\mu) R(\mu, \mu) [N^+(\mu) + N^-(\mu)] + \\ + \lambda(\mu) [N^+(\mu) - N^-(\mu)] = 0, \quad 0 < \mu < \alpha. \end{aligned} \quad (6.6)$$

Considering formulas of Sokhotsky for dispersion function, let us transform the equation (6.6) to the non-uniform Riemann' boundary value problem

$$\begin{aligned} \lambda^+(\mu) [N^+(\mu) + h_{as}(0, \mu)] - \\ - \lambda^-(\mu) [N^-(\mu) + h_{as}(0, \mu)] = 0, \quad 0 < \mu < \alpha. \end{aligned} \quad (6.7)$$

Let us consider the corresponding homogeneous boundary value problem

$$\frac{X^+(\mu)}{X^-(\mu)} = \frac{\lambda^+(\mu)}{\lambda^-(\mu)}, \quad 0 < \mu < \alpha. \quad (6.8)$$

The solution of this problem which is limited and not disappearing in points $z = 0$ and $z = \alpha$ we will take without a derivation

$$X(z) = \frac{1}{z^2} \exp V(z), \quad (6.9)$$

where

$$V(z) = \frac{1}{\pi} \int_0^\alpha \frac{\theta(\mu) - 2\pi}{\mu - z} d\mu, \quad (6.10)$$

where $\theta(\mu) = \arg \lambda^+(\mu)$ is the principal value of the argument, fixed in zero by condition $\theta(0) = 0$.

Let us transform the problem (6.7) by means of homogeneous problem (6.8) to the problem of finding of analytical function on its jump on the cut,

$$\begin{aligned} X^+(\mu)[N^+(\mu) + h_{as}(0, \mu)] = \\ = X^-(\mu)[N^-(\mu) + h_{as}(0, \mu)], \quad 0 < \mu < \alpha. \end{aligned} \quad (6.11)$$

Let us find singularities of boundary condition (6.11). We will return to function $N(z)$, which boundary values in the interval $(0, \alpha)$ enter into the boundary condition (6.7) and which is defined by equality (6.3). We will present this function in the explicit form

$$N(z) = r_0(a)P(z) + r_1(a)\frac{z}{1-az}Q(z) + r_2(a)\left[\left(\frac{z}{1-az}\right)^2 - \beta(a)\right]R(z), \quad (6.12)$$

where

$$\begin{aligned} P_n(z) &= \int_0^\alpha \eta \frac{\Lambda_n(\eta)}{\lambda(\eta)} \rho(\eta) \frac{A(\eta)d\eta}{\eta - z}, \quad n = 0, 1, 2, \\ Q(z) &= \int_0^\alpha \eta \frac{\Lambda_1(\eta)}{\lambda(\eta)} \rho(\eta) \frac{A(\eta)d\eta}{\eta - z} \equiv P_1(z), \\ R(z) &= \int_0^\alpha \eta \frac{\Lambda_2(\eta) - \beta(a)\Lambda_0(\eta)}{\lambda(\eta)} \rho(\eta) \frac{A(\eta)d\eta}{\eta - z} \equiv P_2(z) - \beta(a)P_0(z), \end{aligned}$$

Let us transform equality (6.12), allocating its polar singularity $\frac{z}{1-az}$:

$$\begin{aligned} N(z) &= r_0(a)P_0(z) - r_2(a)\beta(a)[P_2(z) - \beta(a)P_0(z)] + \\ &+ r_1(a)\frac{z}{1-az}P_1(z) + r_2(a)\left(\frac{z}{1-az}\right)^2[P_2(z) - \beta(a)P_0(z)]. \end{aligned} \quad (6.13)$$

Let us write down equality (6.13) on degrees of its polar singularity

$$N(z) = P(z) + r_1(a)\frac{z}{1-az}Q(z) + r_2(a)\left(\frac{z}{1-az}\right)^2R(z), \quad (6.14)$$

where

$$P(z) = [r_0(a) + \beta^2(a)r_2(a)]P_0(z) - r_2(a)\beta(a)P_2(z),$$

$$Q(z) \equiv P_1(z), \quad R(z) = P_2(z) - \beta(a)P_0(z).$$

From expression (6.14) it is visible, that the boundary condition (6.11) has double pole in the point $z = 1/a$. Therefore, multiplying this boundary condition on $(1 - a\mu)^2$, we receive

$$\begin{aligned} X^+(\mu)[M^+(\mu) + (1 - a\mu)^2 h_{as}(0, \mu)] = \\ = X^-(\mu)[M^-(\mu) + (1 - a\mu)^2 h_{as}(0, \mu)], \quad 0 < \mu < \alpha, \end{aligned} \quad (6.15)$$

where

$$M(z) = (1 - az)^2 N(z). \quad (6.16)$$

Considering behaviour of the functions entering into the boundary condition (6.15), we receive the general solution of the corresponding boundary problem

$$M(z) = -(1 - az)^2 h_{as}(0, z) + \frac{C_0 + C_1 z}{X(z)}, \quad (6.17)$$

where C_0 and C_1 are arbitrary constants, though in this equality

$$\begin{aligned} (1 - az)^2 h_{as}(0, z) = \\ = (1 - az)^2 (\varepsilon_n + \varepsilon_T) - (1 - az)z(2U + \omega(a)g_T) - \left[z^2 - \frac{3}{2}(1 - az)^2 \right] (\varepsilon_n - g_T z). \end{aligned}$$

Let us notice, that the solution (6.17) has in infinitely removed point $z = \infty$ a pole of the third order, while function $M(z)$, defined by equality (6.16), has in this point a pole the first order. That the solution (6.17) could be accepted in quality of function $M(z)$, defined by equality (6.16), we will lower order of a pole at the solution (6.17) from three to unit, and then let us equate coefficients at z^m ($m = 1, 0$) in expansion in point vicinities $z = \infty$ the left and right parts of equality (6.17). The last is caused by that these coefficients in both equality parts contain unknown parametres.

Let us expand both the left and right parts of the solution (6.17) in Laurent series in a vicinity of infinitely remote point

$$\begin{aligned}
M_1 z + M_0 + o(1) &= C_1 z^3 + (C_0 - V_1) z^2 + (C_1 U_2 - C_0 V_1) z + \\
&+ (C_1 U_3 - C_0 U_2) - \varepsilon_n + \frac{1}{2} \varepsilon_T + (2a\varepsilon_n - a\varepsilon_T - 2U - g_T \omega(a) - \frac{3}{2} g_T) z + \\
&+ \left[-a^2 \varepsilon_n + \varepsilon_T \left(\frac{a^2}{2} - 1 \right) + 2aU + a(\omega(a) + 3) g_T + \right] z^2 + \\
&+ \left(1 - \frac{3}{2} a^2 \right) g_T z^3 + o(1), \quad z \rightarrow \infty.
\end{aligned} \tag{6.18}$$

At a derivation of equality (6.18) expansion has been used

$$V(z) = \frac{V_1}{z} + \frac{V_2}{z^2} + \dots, \quad z \rightarrow \infty.$$

Here

$$V_n = -\frac{1}{\pi} \int_0^\infty \tau^{n-1} [\theta(\tau) - 2\pi] d\tau, \quad n = 1, 2, \dots.$$

Besides, at the derivation (6.18) one more has been used expansion

$$\exp \left[-V(z) \right] = 1 + \frac{V_1^*}{z} + \frac{V_2^*}{z^2} + \dots, \quad z \rightarrow \infty. \tag{6.19}$$

Coefficients V_n^* are expressed through coefficients V_n with the help of the equality found on the basis of (6.19)

$$V'(z) = \frac{\frac{V_1^*}{z^2} + 2\frac{V_2^*}{z^3} + 3\frac{V_3^*}{z^4} \dots}{1 + \frac{V_1^*}{z} + \frac{V_2^*}{z^2} + \frac{V_3^*}{z^3} \dots}.$$

Really, substituting in the left part of this equality the series for $V(z)$, we receive

$$\begin{aligned}
& - \left(\frac{V_1}{z^2} + 2\frac{V_2}{z^3} + 3\frac{V_3}{z^4} \dots \right) \left(1 + \frac{V_1^*}{z} + \frac{V_2^*}{z^2} + \frac{V_3^*}{z^3} \dots \right) = \\
& = \frac{V_1^*}{z^2} + 2\frac{V_2^*}{z^3} + 3\frac{V_3^*}{z^4} \dots
\end{aligned}$$

From here we find that

$$V_1^* = -V_1, \quad V_2^* = -V_2 + \frac{1}{2}V_1^2,$$

$$V_3^* = -V_3 + V_1V_2 - \frac{1}{6}V_1^3, \dots$$

Equating to zero in the right part of equality (6.18) coefficients at z^3 and z^2 , we find

$$C_1 = g_T \left(\frac{3}{2}a^2 - 1 \right), \quad (6.20)$$

$$\begin{aligned} C_0 = g_T \left[\left(\frac{3}{2}a^2 - 1 \right) V_1 - a(\omega(a) + 3) \right] + \\ + a^2 \varepsilon_n + \left(1 - \frac{a^2}{2} \right) \varepsilon_T - 2aU. \end{aligned} \quad (6.21)$$

Equating now coefficients in (6.18) at the left and on the right at z and z^0 , we find

$$M_1 = C_1 V_2^* - C_0 V_1 + 2a\varepsilon_n - 2U - \left(\omega(a) + \frac{3}{2} \right) g_T, \quad (6.22)$$

$$M_2 = C_1 V_3^* + C_0 V_2^* - \varepsilon_n + \frac{1}{2} \varepsilon_T. \quad (6.23)$$

Substituting in equalities (6.22) and (6.23) coefficients C_0 and C_1 , defined accordingly equalities (6.20) and (6.21), we will receive the equations, from which unequivocally searched ε_n and ε_T . Thus, free parameters C_0 and C_1 from solution $M(z)$ are found unequivocally, and also coefficients of the discrete spectrum are found unequivocally ε_n and ε_T of expansion (6.1). Coefficient of continuous spectrum $A(\eta)$ is searched on the basis of formulas Sokhotsky, applied to functions $M(z)$, the defined equalities (6.16) and (6.17)

$$M_+(\mu) - M_-(\mu) = 2\pi i \mu (1 - a\mu)^2 \frac{R(\mu, \mu)}{\lambda(\mu)} \rho(\mu) A(\mu)$$

and

$$M_+(\mu) - M_-(\mu) = (C_0 + C_1 \mu) \left(\frac{1}{X^+(\mu)} - \frac{1}{X^-(\mu)} \right).$$

From these equalities we find

$$\frac{\eta A(\eta) \rho(\eta)}{\lambda(\eta)} = \frac{C_0 + C_1 \eta}{2\pi i (1 - a\eta)^2 R(\eta, \eta)} \left(\frac{1}{X^+(\eta)} - \frac{1}{X^-(\eta)} \right). \quad (7.24)$$

So, all coefficients of expansion (6.1) are established. On construction, expansion (6.1) satisfies to boundary conditions (4.3) and (4.4). That fact, that expansion (6.1) satisfies to the equation (4.2), it is checked directly.

Uniqueness of decomposition (6.1) is proved by a method from the opposite. The theorem is proved.

7. Temperature jump and weak evaporation (condensation)

Let us return to the decision of the put physical problems. Coefficient of continuous spectrum (6.24) we will substitute at first in equality (6.3) for functions $N(z)$, and then we will take advantage of equality (6.16), which let us present in the form of the sum

$$M(z) = C_0 K(z) + C_1 L(z), \quad (7.1)$$

where

$$K(z) = \frac{(1 - az)^2}{2\pi i} \int_0^\alpha \frac{R(\eta, z)}{(1 - a\eta)^2 R(\eta, \eta)} \left[\frac{1}{X^+(\eta)} - \frac{1}{X^-(\eta)} \right] \frac{d\eta}{\eta - z}, \quad (7.2)$$

and

$$L(z) = \frac{(1 - az)^2}{2\pi i} \int_0^\alpha \frac{\eta R(\eta, z)}{(1 - a\eta)^2 R(\eta, \eta)} \left[\frac{1}{X^+(\eta)} - \frac{1}{X^-(\eta)} \right] \frac{d\eta}{\eta - z}. \quad (7.3)$$

According to (7.1) we have

$$M(z) = (C_0 K_1 + C_1 L_1) + (C_0 K_0 + C_1 L_0) + o(1), \quad z \rightarrow \infty, \quad (7.4)$$

i.e.

$$M_1 = C_0 K_1 + C_1 L_1, \quad M_0 = C_0 K_0 + C_1 L_0. \quad (7.5)$$

The found coefficients M_1 and M_2 we will substitute in equalities (6.22) and (6.23), then we will replace in them C_0 and C_1 it agree (6.20) and (6.21). We receive system from two equations rather ε_T and ε_n :

$$\begin{aligned} & -\left[a + \left(1 - \frac{a^2}{2}\right)(V_1 + K_1)\right]\varepsilon_T + [2a - a^2(V_1 + K_1)]\varepsilon_n = \\ & = 2U[1 - a(V_1 + K_1)] + g_T\left[\frac{3}{2} + \omega + \left(\frac{3}{2} - 1\right)(L_1 - \right. \\ & \quad \left. - V_2^* + V_1^2 + V_1K_1) - (3a - \omega(a))(V_1K_1)\right] \end{aligned} \quad (7.6)$$

and

$$\begin{aligned} & \left[\frac{1}{2} + \left(1 - \frac{a^2}{2}\right)(V_2^* - K_0)\right]\varepsilon_T + [-1 + a^2(V_2^* - K_0)]\varepsilon_n = \\ & = 2aU(V_2^* - K_0) + g_T\left[\left(\frac{3}{2}a^2 - 1\right)(V_1K_0 - V_1V_2 - \right. \\ & \quad \left. - V_3 + L_0) + (3a + \omega(a))(V_2 + K_0)\right]. \end{aligned} \quad (7.7)$$

The basic determinant of this system is equal

$$\Delta = V_1 + K_1 - 2a(V_2^* - K_0).$$

Let us bring expressions for other determinants

$$\begin{aligned} \Delta_{T,U} &= -1 + a(V_1 + K_1) - a^2(V_2^* - K_0), \\ \Delta_{n,U} &= -\frac{1}{2} + \frac{1}{2}a(V_1 + K_1) - \left(1 + \frac{a^2}{2}\right)(V_2^* - K_0), \\ \Delta_{T,g_T} &= [1 - a^2(V_2^* - K_0)]\left[\left(\frac{3}{2}a^2 - 1\right)(V_2^* - L_1) - \omega(a) - \frac{3}{2}\right] + \\ & + \left[\left(\frac{3}{2}a^2 - 1\right)V_1 - 3a - \omega(a)\right][2a(V_2^* - K_0) - V_1 - K_1] + \\ & + \left(\frac{3}{2}a^2 - 1\right)(V_3^* - L_0)[2a - a^2(V_1 + K_1)], \\ \Delta_{n,g_T} &= -\left[a + \left(1 - \frac{a^2}{2}\right)(V_1 + K_1)\right]\left[(3a + \omega(a))(V_2^* - K_0) - \left(\frac{3}{2}a^2 - 1\right) \times \right. \\ & \quad \left. \times (V_3^* - L_0 + V_1V_2^+ - V_1K_0)\right] - \left[\frac{1}{2} + \left(1 - \frac{a^2}{2}\right)(V_2^* - K_0)\right] \times \end{aligned}$$

$$\times \left[\frac{3}{2} + \omega(a) + \left(\frac{3}{2}a^2 - 1 \right) (L_1 - V_2^* + V_1^2 + V_1 K_1) - (3a + \omega(a))(V_1 + K_1) \right].$$

We will write down the solution of system of the equations (7.6) and (7.7) in the form

$$\varepsilon_T = 2U \frac{\Delta_{T,U}}{\Delta} + g_T \frac{\Delta_{T,g_T}}{\Delta}, \quad (7.8)$$

and

$$\varepsilon_n = 2U \frac{\Delta_{n,U}}{\Delta} + g_T \frac{\Delta_{n,g_T}}{\Delta}. \quad (7.9)$$

8. Conclusion

In the present work the analytical solution of boundary problems for the one-dimensional kinetic equation with frequency of collisions of molecules, affine depending on the module molecular velocity [1] is considered. The solution of the generalized Smoluchowsky' problem about temperature jump and weak evaporation (condensation) is considered.

The theorem of expansion of the solution of the Smoluchowsky' problem on eigenfunctions of the corresponding characteristic equation is proved

.

REFERENCES

- [1] *Bugrimov A.L., Latyshev A.V., Yushkanov A.A.* The kinetic one-dimensional equation with frequency of collisions, affine depending on the module molecular velocity// arXiv:1403.2068v1 [math-ph] 9 Mar 2014, 20pp.
- [2] *Latyshev A.V., Yushkanov A.A.* Kinetic equations type Williams and their exact solutions. Monograph. M.: MGOU (Moscow State Regional University), 2004, 271 p.

- [3] *Latyshev A.V.* Application of Case' method to the solution of linear kinetic BGK equation in a problem about temperature jump// Appl. math. and mechanics. 1990. V. 54. Issue 4. P. 581–586. [russian]
- [4] *Latyshev A.V., Yushkanov A.A.* Boundary problems for model Boltzmann equation with frequency proportional to velocity of moleculs// Izvestiya Russian Academy of Science. Ser. Mechanika, Fluid and Gas (Russian "Fluids Dynamics"). 1993. №6. 143-155 pp. [russian]
- [5] *Latyshev A.V., Yushkanov A.A.* Analytical solution of the problem about strong evaporation (condensation)// Izvestiya Russian Academy of Science. Ser. Mechanika, Fluid and Gas (Russian "Fluids Dynamics"). 1993. №6. 143-155 pp. [russian]
- [6] *Cassell J.S., Williams M.M.R.* An exact solution of the temperature slip problem in rarefied gases// Transport Theory and Statist. Physics, 2(1), 81–90 (1972).
- [7] *Latyshev A.V., Yushkanov A.A.* Temperature jump and weak evaporatuion in molecular gases// J. of experim. and theor. physics. 1998. V. 114. Issue. 3(9). P. 956–971. [russian]
- [8] *Latyshev A.V., Yushkanov A.A.* The Smoluchowski problem in polyatomic gases// Letters in J. of Tech. Phys. 1998. V. 24. №17. P. 85–90. [russian]
- [9] *Latyshev A.V., Yushkanov A.A.* Analytic solutions of boundary value problem for model kinetic equatins// Math. Models of Non-Linear Excitations, Transfer, Dynamics, and Control in Condensed Systems and Other Media. Edited by L.A. Uvarova and A.V. Latyshev. Kluwer Academic. New York–Moscow. 2001. P. 17–24.

- [10] *Latyshev A.V., Yushkanov A.A.* Smolukhowski problem for degenerate Bose gases// Theoretical and Mathematical Physics. Springer New York. Vol. 155, №3, June, 2008, pp. 936 – 948.
- [11] *Latyshev A.V., Yushkanov A.A.* Temperature jump in degenerate quantum gases in the presence of the Bose–Einstein condensate // Theor. and Mathem. Phys. 2010. V. 162(1). P. 95–105 [russian]
- [12] *Latyshev A.V., Yushkanov A.A.* Temperature jump in degenerate quantum gases with the Bogoliubov excitation energy and in the presence of the Bose–Einstein condensate, Theoret. and Math. Phys., 165:1 (2010), 1358–1370.
- [13] *Latyshev A.V., Yushkanov A.A.* Smoluchowski problem for electrons in metal// Theor. and Mathem. Phys. 2005, январь, Т. 142. №1. С. 93–111 [russian]
- [14] *Latyshev A.V., Yushkanov A.A.* Smoluchowski problem for metals with mirror-diffusive boundary conditions //Theoretical and Mathematical Physics October 2009, Volume 161, Issue 1, pp. 1403-1414.
- [15] *Latyshev A.V., Yushkanov A.A.* Boundary value problems for quantum gases. Monograph M.: MGOU, 2012, 266 p.[russian]
- [16] *Cercignani C., Frezzoti A.* Linearized analysis of a one-speed B.G.K. model in the case of strong condensation// Bulgarian Academy of sci. theor. appl. mech. Sofia. 1988. V.XIX. №3. 19-23 P.
- [17] *Latyshev A.V., Yushkanov A.A.* Analytical solution of one-dimensional problem about moderate strong evaporation (and condensation) in half-space// Appl. mech. and tech. physics. 1993. №1. 102-109 p. [russian]

- [18] *Vladimirov V.S., Zharinov V.V.* Equations of mathematical physics. M.: Fizmatlit. 2000. 399 c.[russian]
- [19] *Gakhov F.D.* Boundary value problems. M.: Nauka. 640 p.[russian]